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Guidance Law for an Air-Breathing Launch Vehicle Using Predictive Control Concept

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I. Introduction

AN air-breathing launch vehicle (ABLV) is a relatively new concept in the space transportation system that can operate from conventional runways like an aircraft, yet reach hypersonic velocities. The use of air-breathing hypersonic propulsion technology promises an order of magnitude reduction in the cost of placing payloads in low Earth orbit (LEO) [1]. The most important feature of air-breathing propulsion is high specific impulse [2], I_{sp} values (2000–4000 s) compared to conventional rockets. Such a vehicle must transit from launch to hypersonic Mach numbers at altitudes between 20 and 30 km for ensuring the best performance of the air-breathing engine. Potential applications of this new launch vehicle include delivering large mass payloads into LEO, intercontinental passenger transportation, and a wide range of defense missions. Trajectory optimization and guidance are two key elements of any mission design to maximize the payload into orbit. Current practice in rocket launch vehicles is to use a preprogrammed open loop pitch and yaw profiles for the atmospheric portion of flight and then use the linear tangent steering law for the vacuum portion of flight. An ABLV dwells in the atmosphere for a much greater portion of the trajectory than the conventional counterparts. Real time computation of the optimal trajectory for an ABLV is complicated due to the fact that the air-breathing engine performance (thrust and specific impulse, I_{sp}) is highly sensitive to flight parameters like Mach number M, angle of attack α, and fuel equivalence ratio φ [3]. However it is possible to achieve good results by offline generation of the optimal trajectory using powerful numerical techniques and employing an efficient trajectory tracking guidance law for generating the steering commands online.

The numerical approach to optimal guidance typically employs nonlinear programming [4,5]. To be useful as a feedback guidance solution, it is essential that these approaches converge quickly and reliably at each instant when the solution is updated during the flight. Ping Lu [6] developed a simple and elegant guidance law for trajectory tracking of a launch vehicle. This guidance law is based on a continuous time predictive control concept derived in [7]. The present study is an extended application and validation of the above work for trajectory tracking in the ascent phase of a generic air-breathing launch vehicle. A guidance law was implemented by incorporating appropriate trajectory constraints and system response was predicted as a function of the current control. The controller was designed to minimize a weighted difference between desired and predicted responses for the mission under consideration. A judicious choice of the weightings enabled an efficient trajectory tracking in terms of required accuracy as well as desired damping requirements. The application of the guidance law [6] together with the continuous time predictive control law [7] has proven to be very effective in tracking the trajectory of an ABLV even in the presence of aerodynamic dispersions and engine off-nominal performances. Appropriate choices of saturation bounds demonstrated the capability of the guidance law to enforce the constraints on dynamic pressure (q), angle of attack (α), and aerodynamic load ($q\alpha$). This analysis suggests that the present guidance scheme [6,7] may be ideal for a reentry phase of the ABLV mission as well. This methodology can be used to define a unified guidance scheme from liftoff to touchdown for reusable launch vehicle (RLV) missions employing both rocket propulsion as well as air-breathing propulsion.

This Note is organized as follows: the vehicle model and generation of optimal trajectory is explained in Sec. II. Section III reviews in brief the prediction of system response and the tracking errors. A brief introduction to the continuous time predictive control law is given in Sec. IV. Implementation of the trajectory tracking guidance law for an ABLV is described in Sec. V, and simulation results of the guidance law are presented in Sec. VI. Finally, conclusions are given in Sec. VII.

II. Vehicle Model and Optimal Trajectory Synthesis

A conceptual ABLV is used for the study. The system consists of a rocket based multipropulsion cycle engine whose most important feature is the high specific impulse (2000–4000 s) during the flight within the atmosphere from Mach 1.5 till 7. This launch vehicle has a liftoff mass of about 176 t and consumes about 16 t fuel during the air-breathing ascent phase to Mach 7. The point mass equations of motion of the vehicle in a vertical plane over a nonrotating, spherical Earth are given by

$$\dot{v} = -g\sin\gamma + \frac{(T\cos\alpha - D)}{m} \tag{1}$$

$$\dot{\gamma} = \left(\frac{v}{r} - \frac{g}{v}\right)\cos\gamma + \frac{(L + T\sin\alpha)}{mv} \tag{2}$$

$$\dot{r} = v\sin\gamma \tag{3}$$

$$\dot{R} = v\cos\gamma \tag{4}$$

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where v is the vehicle velocity, g the acceleration due to gravity, r the radial distance of the vehicle from the center of Earth, m the vehicle mass, γ the flight path angle, and α the angle of attack. The lift and the drag coefficients are given by

$$L = qS_{\text{ref}}C_L\alpha \tag{5}$$

$$D = qS_{\text{ref}}C_D \tag{6}$$

Thrust, lift, and drag forces are given by T, L, and D. Lift coefficient is given by C_L and drag coefficient by C_D. Indian standard atmosphere is used where atmospheric properties such as density, pressure, and temperature are stored as a function of altitude. Density (ρ) will be used for computation of dynamic pressure q ($q = \frac{1}{2}\rho V_r^2$), which will be required by lift and drag equations. V_r is the relative velocity and S_{ref} is the reference area. Pressure will be used for thrust correction and temperature for the computation of speed of sound. Thrust as a function of the Mach number, and fuel equivalence ratio φ and aerodynamic coefficients C_L and C_D as a function of the Mach number and angle of attack α are the inputs. Gravity is computed using the expression $g = \mu/r^2$, where μ is the gravitational parameter, and r_0 is the radius of Earth.

Trajectory optimization of an air-breathing launch vehicle is a challenging task, because the performance of the engine is highly sensitive to the flight path. In the present study, reference optimal trajectory is generated offline using the nonlinear programming method (NLP). The NLP problem is solved using the sequential quadratic programming (SQP) method. The trajectory is optimized for maximizing the payload satisfying the following inequality constraints:

$$q \leq 62 \text{ kPa} \tag{7}$$

$$1 \leq \alpha \leq 5 \text{ deg} \tag{8}$$

$$q\alpha \leq 5400 \text{ Pa} \cdot \text{rad} \tag{9}$$

where $q\alpha$ represents the aerodynamic load acting on the vehicle. The constraints on q and $q\alpha$ have to be satisfied to ensure controllability and maintain the structural integrity of the vehicle. The constraint on α is imposed to ensure that the air-breathing engine performance is assessed in the required range specified in the mission. The terminal condition imposed on the optimal trajectory is that the burnout Mach number should be above 7.

The trajectory tracking problem is to determine the control commands that would guide the vehicle to track the nominal trajectory in the presence of off-nominal engine performance and aerodynamic dispersions. Pitch angle θ is used as the control command to steer the vehicle along the reference nominal trajectory, and out of plane motion is not considered.

III. Prediction of System Response

This section gives a brief introduction of the continuous time predictive control law used in the trajectory tracking guidance of launch vehicles. In the predictive control approach, the response of the system is predicted as a function of the control. The controller is designed to minimize a function of the weighted difference between the predicted and desired responses. Consider the nonlinear dynamic system of the general form

$$\dot{x}_1 = f_1(x) \tag{10}$$

$$\dot{x}_2 = f_2(x) + g_2(x, u) \tag{11}$$

where $x = [x_1 x_2]^T$ is the state vector, u is the control vector, and the functions f_1, f_2, and g_2 are continuously differentiable nonlinear functions. The control vector u is bounded by

$$Li[x(t)] \leq u(t) \leq Ui[x(t)] \tag{12}$$

where the bounds Li and Ui are specified and allowed to be state dependent. Suppose that a desired trajectory $x^*(t)$ is already known and that satisfies Eqs. (10) and (11) with a corresponding control $r^*(t) \in U = \{u(t) \in Li[x(t)] \leq u(t) \leq Ui[x(t)]\}$. If at an arbitrary instant $t \in [0, t_f]$, $x(t)$ is known then the current control $u(t)$ determines the system response in the immediate future, say at $t + h$, where h is a small time increment.

Assuming that $\ddot{x}_1(t)$ and $\dot{x}_2(t)$ depend on $u(t)$ explicitly, the influence of $u(t)$ on x_1 at $(t + h)$ may be predicted by a second order Taylor series expansion at t and on x_2 at $(t + h)$ by a first order expansion. Let the predicted responses at $t + h$ be $x_1(t + h)$ and $x_2(t + h)$, and the desired responses $x_1^*(t + h)$, and $x_2^*(t + h)$, then the tracking error at $(t + h)$ can be approximated by

$$e_1(t+h) = x_1(t+h) - x_1^*(t+h) \approx e_1(t) + h\dot{e}_1(t) + 0.5h^2[F_{11}(x)f_1(x) + F_{12}(x)f_2(x) + F_{12}(x)g_2(x,u) - \ddot{x}_1^*(t)] \tag{13}$$

$$e_2(t+h) = x_2(t+h) - x_2^*(t+h) = e_2(t) + h[f_2(x) + g_2(x,u) - \dot{x}_2^*] \tag{14}$$

where $F_{11} = \partial f_1[x(t)]/\partial x_1$ and $F_{12} = \partial f_1[x(t)]/\partial x_2$.

IV. Continuous Time Predictive Control Law

To find the control $u(t)$ that improves the tracking accuracy at the next instant, consider a pointwise minimization of the performance index that penalizes the tracking error at $t + h$, and the current control expenditure,

$$\min_{u(t)\in U} J = \tfrac{1}{2}e_1^T(t+h)Q_1(t)e_1(t+h) + \tfrac{1}{2}e_2^T(t+h)Q_2(t)e_2(t+h) + \tfrac{1}{2}[u(t) - r^*(t)]^T R(t)[u(t) - r^*(t)] \tag{15}$$

The controller minimizes a weighted function of tracking errors, where Q_1 and Q_2 (weightings) are positive semidefinite matrices and R (weighting) positive definite. Applying the necessary condition for optimality $\partial J/\partial U = 0$ leads to a continuous time control law $u(t)$ as follows:

$$u(t) = r^*(t) - hR^{-1}\left(0.5h\left[F_{12}(x)\frac{\partial g_2(x,u)}{\partial u}\right]^T Q_1\left\{e_1 + h\dot{e}_1 + 0.5h^2\left[F_{11}(x)f_1(x) + F_{12}(x)f_2(x) + F_{12}(x)g_2(x,u) - \ddot{x}_1^*\right]\right\} + \left[\frac{\partial g_2(x,u)}{\partial u}\right]^T Q_2\left\{e_2(t) + h\left[f_2(x) + g_2(x,u) - \dot{x}_2^*\right]\right\}\right) = \eta(u) \tag{16}$$

$$u(t) = r^*(t) - hR^{-1}N(x, x^*, u) \tag{17}$$

The nonlinear vector term N is defined by comparing Eq. (17) with Eq. (16). This nonlinear vector term involves the weightings Q_1, Q_2 and the tracking errors.

Applying the vector saturation function s to handle the bounds on the control we can rewrite the control law as

$$u(t) = s[r^*(t) - hR^{-1}N(x, x^*, u)] \tag{18}$$

From Eqs. (16) and (18), the control law can be written as

$$u(t) = s[\eta(u)] \tag{19}$$

Equation (18) defines a continuous implicit control law for $u(t)$ with a feedforward term $r^*(t)$ and a feedback part associated with the tracking errors. The schematic diagram for continuous time implementation of the control law is shown in Fig. 1.

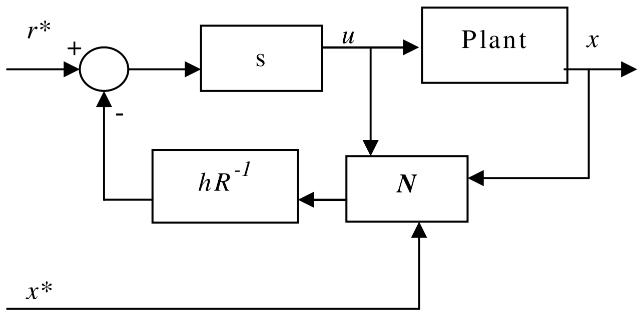

Fig. 1 Continuous time implementation of the control law.

V. Implementation Trajectory Tracking Guidance Law for ABLV

In this section the preceding approach is applied in simulations to track a nominal trajectory for an air-breathing launch vehicle. The guidance algorithm uses pitch angle as the control variable. The pitch angle affects the dynamics of the system through angle of attack α and flight path angle γ. The algorithm generates the necessary steering commands for effectively tracking the reference altitude and climb rate profiles. For better numerical conditions the following dimensionless variables are used:

$$t = \frac{t}{\sqrt{r_0/g_0}} \qquad Y = \frac{r}{r_0} \qquad V = \frac{v}{\sqrt{g_0 r_0}} \qquad A_T = \frac{T}{m_0 g_0}$$
$$A_D = \frac{D}{m_0 g_0} \qquad A_L = \frac{L}{m_0 g_0} \tag{20}$$

where m_0 is the vehicle initial mass, r_0 is the Earth radius, $g_0 = \mu/r_0$, μ is the gravitational parameter, r is the current radial distance, v the velocity, T the thrust, D the drag, and L the lift. In addition the climb rate $Z = \mathrm{d}Y/\mathrm{d}\tau$ is used in the place of the flight path angle γ. This has the advantage that the feedback control law will depend on Z, which is easier to measure than γ. Thus the system equations take the following form in the new dimensionless variables:

$$Y' = Z \tag{21}$$

$$Z' = (V^2/Y) - (1/Y^2) - (Z^2/Y) + [A_T \cos(\theta - \gamma) - A_D](Z/V) + [A_T \sin(\theta - \gamma) + A_L]\sqrt{1 - (Z/V)^2} \tag{22}$$

$$V' = A_T \cos(\theta - \gamma) - A_D - (Z/VY^2) \tag{23}$$

where the prime (′) stands for differentiation with respect to τ. The throttle is set at a maximum value and hence it is difficult to achieve feedback velocity regulation effectively. Therefore, we will be concentrating on flight path control by regulating two of the state variables, $x_1 = Y$ (altitude) and $x_2 = Z$ (climb rate), that is, Eqs. (21) and (22). By using the dimensionless variables the guidance law by Eq. (19) for θ can be written as

$$\theta(\tau) = s(\theta^*(\tau) - hR^{-1}\{0.5hQ_Y G_{21}[\Delta Y + h\Delta Z + 0.5h^2(f_{22} + g_{22} - Z'^*)] + G_{21}Q_Z[\Delta Z + h(f_{22} + g_{22} - Z'^*)]\}) \tag{24}$$

In Eq. (24), s is the saturation function and

$$f_{22} = \left(\frac{V^2}{Y}\right) - \left(\frac{1}{Y^2}\right) - \left(\frac{Z^2}{Y}\right)$$

$$g_{22} = [A_T \cos(\theta - \gamma) - A_D]\left(\frac{Z}{V}\right) + [A_T \sin(\theta - \gamma) + A_L]\sqrt{1 - \left(\frac{Z}{V}\right)^2}$$

$$G_{21} = -\left[A_T \sin(\theta - \gamma) + \frac{\partial A_D}{\partial \alpha}\right]\left(\frac{Z}{V}\right) + \left[A_T \cos(\theta - \gamma) + \frac{\partial A_L}{\partial \alpha}\right]\sqrt{1 - \left(\frac{Z}{V}\right)}$$

where

$$\frac{\partial A_L}{\partial \alpha} = \frac{\rho v^2 S_{\mathrm{ref}}}{2mg_0}\frac{\partial C_L}{\partial \alpha}, \qquad \frac{\partial A_D}{\partial \alpha} = \frac{\rho v^2 S_{\mathrm{ref}}}{2mg_0}\frac{\partial C_D}{\partial \alpha}$$

The guidance law can be written as

$$\theta(\tau) = s[\theta^*(\tau) - hR^{-1}N] \tag{25}$$

The specific form of N is defined by comparing Eq. (25) with Eq. (24). The control command θ is obtained iteratively from Eq. (24). A_D and A_L depend on θ through α.

Function of the saturator s in Eq. (24) is to ensure that the constraints on α and $q\alpha$ are satisfied. The requirements Eqs. (8) and (9) are ensured by choosing the saturation bounds as follows in the definition of the saturator s.

$$U = \min\left(5 \ \mathrm{deg} + \gamma, \frac{5400}{q} + \gamma\right) \tag{26}$$

$$L = \max\left(1 \ \mathrm{deg} + \gamma, \frac{1080}{q} + \gamma\right) \tag{27}$$

Then θ obtained from Eq. (24) guarantees the satisfaction of all the constraints. The following weightings are chosen in the guidance law for effectively tracking the specified flight path $[Y^*(t)Z^*(t)]$,

$$Q_1 = Q_Y = 2, \qquad Q_2 = Q_Z = 0.25, \qquad h = 0.5$$
$$R(\tau) = 1 - 0.99(\tau/\tau_f)$$

A. Selection of Step Size *h*

The selection of step size h was found to be very important from the point of view of rate of convergence as well as the accuracy of tracking. The initial value of h was selected as 0.5. The size of h was halved if convergence had not been achieved within a fixed number of iterations. This process was continued until convergence occurred. The largest value of h used in the simulation was 0.5 and the smallest value was 0.03125.

B. Criterion for Iteration Convergence

The typical number of iterations required at a point was 2–4 though the maximum allowed number for convergence was 15. The criterion for convergence was selected to achieve the desired state (altitude) with minimum error.

Table 1 Dispersions in terminal conditions

Dispersion cases	Δh_f, km	ΔV_f, ms^{-1}	$\Delta \gamma_f$, deg	M_f	m_f, t
Case 1: $T = 0.98T^*$, $I_{\mathrm{sp}} = 0.98I^*_{\mathrm{sp}}$	0.097	−0.84	0.16	7.02	157.7
Case 2: $T = 1.02T^*$, $I_{\mathrm{sp}} = 1.02I^*_{\mathrm{sp}}$	0.105	−0.022	0.16	7.03	161.8
Case 3: $C_L = 0.7C^*_L$, $C_D = 1.3C^*_D$	0.094	−0.514	0.26	7.02	158.1
Case 4: $C_L = 1.3C^*_L$, $C_D = 0.7C^*_D$	0.100	0.888	0.11	7.03	161.6

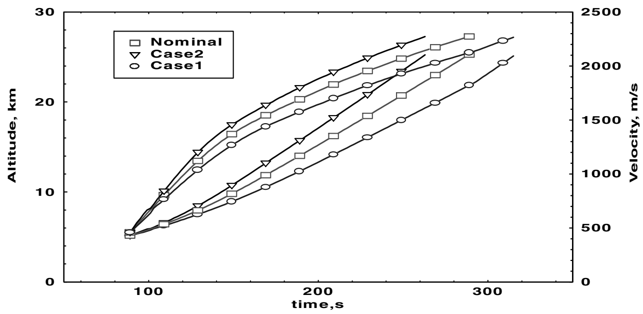

Fig. 2 Altitude and velocity profiles for cases 1 and 2.

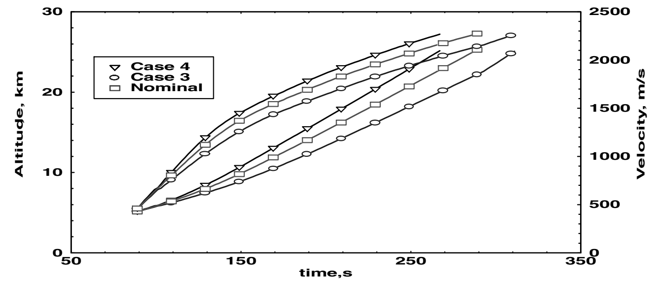

Fig. 3 Altitude and velocity profiles for cases 3 and 4.

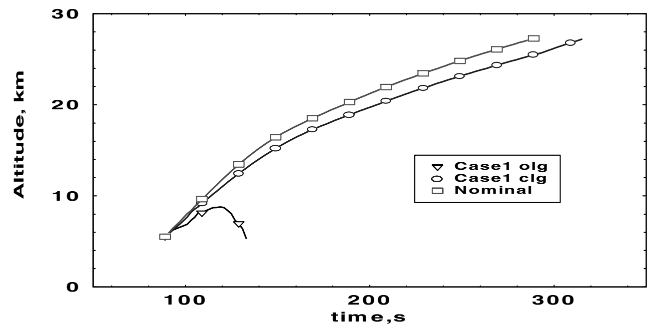

Fig. 4 Altitude profiles for cases 1, olg vs clg.

VI. Validation of Guidance Law

The trajectory tracking guidance law implemented in Sec. V was applied to a generic air-breathing launch vehicle to track the nominal trajectory. The algorithm was validated by simulating both nominal and off-nominal vehicle performances and dispersions in aerodynamic coefficients.

A. Initial Conditions for Guidance

The initial values used in the simulation are as follows: altitude = 5.450 km; velocity = 428.8 m/s; flight path angle = 28.77 deg; mass = 176 t; and pitch angle = 31 deg.

B. Desired End Conditions

The desired end conditions are as follows: altitude $h_f =$ 27.284 km; velocity $V_f = 2125.5$ m/s; flight path angle $\gamma_f =$ 1.646 deg; Mach number $M_f = 7.03$; and mass $m_f = 160$ t.

C. Off-Nominal Cases Studied

The guidance algorithm was validated for various types of off-nominal conditions. The off-nominal cases studied were case 1: thrust $= 0.98T^*$, $I_{sp} = 0.98I_{sp}^*$; case 2: thrust $= 1.02T^*$, $I_{sp} = 1.02I_{sp}^*$ (engine off-nominal performance); case 3: $C_L = 0.7C_L^*$, $C_D = 1.3C_D^*$; case 4: $C_L = 1.3C_L^*$, $C_D = 0.7C_D^*$ (aerodynamic dispersion), with $*$ standing for the nominal value.

D. Analysis of Simulation Results

Dispersions in achieved terminal conditions are given in Table 1. Guidance command to shut off the engine was issued once the target Mach number was reached. Maximum dispersion in achieved altitude was 0.1 km. The maximum velocity dispersion was less than 1 m/s and the maximum error in the flight path angle was 0.26 deg. Achieved end conditions in altitude, velocity, flight path angle, Mach number, and mass for all the above cases were well within the mission specifications. This clearly establishes the efficient trajectory tracking capability of the guidance law. Variations of altitude and velocity for the above dispersion cases are shown in Figs. 2 and 3.

Figure 4 shows the comparison of open loop guidance (olg) and closed loop guidance (clg, using the nonlinear tracking guidance law) performances for thrust dispersion cases 1 and 2. If the vehicle is steered using the nominal open loop pitch steering program obtained in the trajectory optimization problem, the vehicle would impact on ground shortly after takeoff.

VII. Conclusion

A nonlinear tracking guidance algorithm is implemented and validated for the atmospheric ascent phase of a generic air-breathing launch vehicle. The guidance algorithm works based on a nonlinear continuous time predictive control concept. The guidance law is validated for nominal and off-nominal cases. The guidance law generates the pitch steering commands to track the reference trajectory efficiently. Maximum dispersions in achieved target conditions are minimum for thrust and aerodynamic dispersion cases. Maximum dispersions in altitude, velocity, and flight path angle are 0.1 km, 0.88 m/s, and 0.26 deg, respectively. The guidance law also ensures that the constraints on the angle of attack, dynamic pressure, and aerodynamic load are always met without affecting the fuel optimality. The work suggests that the present scheme can be used as a unified guidance scheme for reusable launch vehicle missions from liftoff to touchdown irrespective of any propulsion options, rocket or air breathing.

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